

Examples of calculation methods for each year group and the progression between each method.

This calculation policy has been created to meet the expectations of the National Curriculum 2014, but most importantly the learning needs of our children at Dane Bank. The methods chosen match the national curriculum but have also been specifically selected after consideration of our children's learning styles. This policy has been largely adapted from the White Rose Maths Hub Calculation Policy with further material added. It is a working document and will be revised and amended as necessary.

## Age Expectations

The policy has been organised by year group, considering the National Curriculum 2014 expectations. The new curriculum focuses on skills and mastery and is not about moving children on to the next method as soon as they can do the one before. Working and more complex and richer problems rather than new methods will support this 'mastering' of maths. However, some children will be working at levels well above their age and will require the introduction of new methods.

## Mental Methods

The written methods in this document are important but they by no means replace the superb mental methods we have developed. As children become more mature and confident with their calculation, they need to start following these 4 steps when approaching problems:

$$
\begin{aligned}
& \text { 1) Can I solve it in my } \\
& \text { head and use a } \\
& \text { mental method? } \\
&
\end{aligned}
$$

3) Do I need to use a written method to solve this problem?
4) Do I need to use a calculator to get an answer?

1. EYFS - Y6 calculation policy for addition, with examples of CPA
2. EYFS - Y6 calculation policy for subtraction, with examples of CPA
3. EYFS - Y6 calculation policy for multiplication, with examples of CPA
4. EYFS - Y6 calculation policy for division with examples of CPA
5. Glossary of mathematical vocabulary


Although this policy sets out the main methods of mental and written calculations to be taught, it has been appended with a list of documents and resources which contain effective practice teaching ideas aimed at informing and enhancing teaching across all the primary phases. These are intended to sit alongside the school's calculation policy, and for teachers to use in conjunction with the policy.

1. NCETM's Mastery Booklets (Y1-6)
2. NCETM's Professional development material
3. NCETM's Calculation Guidance document (published October 2015)
4. White Rose Maths' Schemes of Learning (2017 onwards)
5. Numicon Handbooks
6. Dane Bank Primary School KIRFs
7. GLOW Maths Hub "Enabling Environments" (EYFS)
8. GLOW Maths Hub Mastery Scheme of Work
9. Fractions calculation guidance (Manor Green Primary School)

## Addition in EXFS

Maths for young children should be meaningful. Where possible, concepts should be taught in the context of real life.

| GUIDANCE / MODELS AND IMAGES | KEY VOCABULARY |
| :---: | :---: |
| If available, Numicon shapes are introduced straight away and be used to: <br> - identify 1 more/less <br> - combine pieces to add <br> - find number bonds <br> - add without counting <br> Children can record this by printing or drawing around Numicon pieces. <br> Children can begin to combine groups of objects using concrete apparatus: <br> Construct number sentences verbally or using cards to go with practical activities. <br> Children are encouraged to read number sentences aloud in different ways "Three add two equals 5 " " 5 is equal to three and two" " 5 is the same as three and two" <br> Children make a record in pictures, words or symbols of addition activities. <br> Solve simple problems using fingers $\begin{aligned} & 5+1=6 \end{aligned}$ <br> Number tracks can be introduced to count up on and to find one more: $\square$1 2 3 4 5 What is 1 more than 4? 1 more than 13 ? <br> Number lines can be used alongside number tracks and practical $\qquad$ apparatus to solve addition calculations and word problems: <br> Children will need opportunities to look at and talk about different models and images as they move between representations. | Games and songs can be useful way to begin using vocabulary involved in addition i.e. <br> One elephant went out to play <br> plus <br> estimate <br> add <br> more <br> and <br> sum <br> total <br> make <br> altogether <br> score <br> double <br> one more, two more, ten more... <br> how many more make? <br> How many more is ... than ...? <br> same as |

Addition

Focus: Adding with numbers up to 20
Children should use number lines (with the numbers on) to add by counting in ones. Starting with the greatest number and counting on the smaller number.
$6+3=9$


As well as using a numberline, children in Year 1 need to:

- Use a variety of equipment to solve addition problems, including counting equipment, everyday objects, number tracks etc.
- Read and write the addition (+) and equals (=) sign and use them in number sentences.
- Solve addition number sentences and missing number problems: $7+4=$ ? , $1+2+$ $1=?, ?+?=9$ etc.
- Use bead strings or bead bars to visualise bridging through 10 s e.g. $8+5=$ can be solved by counting on 2 then counting on 3.


Key Vocabulary
Add, more, plus, and, make, altogether, total, equal to, equals, the same as, double, most, count on, numberline

Key Skills

- Reading and writing numbers to 100 in numerals.
- Writing numbers to 20 in words including correct spelling.
- Counting to and across 100 in ones.
- Counting in multiples of 2,5 and 10.
- Solving simple one step addition problems: using objects, numberlines and images to support.

| Objective \& Strategy | Concrete | Pictorial | Abstract |
| :---: | :---: | :---: | :---: |
| Combining two parts to make a whole: part- whole model | Use part part whole model. <br> Use cubes to add two numbers together as a group or in a bar. |  | $4+3=7$ $10=6+4$ <br> Use the part-part whole diagram as shown above to move into the abstract. |
| Starting at the bigger number and counting on | Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer. | $12+5=17$ <br> Start at the larger number on the number line and count on in ones or in one jump to find the answer. | $5+12=17$ <br> Place the larger number in your head and count on the smaller number to find your answer. |
| Regrouping to make 10. <br> This is an essential skill for column addition later. | $6+5=11$ <br> Start with the bigger number and use the smaller number to make 10. <br> Use ten frames. | $3+9=$ <br> Use pictures or a number line. Regroup or partition the smaller number using the part part whole model to make 10. | $7+4=11$ <br> If I am at seven, how many more do I need to make 10 . How many more do I add on now? |
| Represent \& use number bonds and related subtraction facts within 20 | 2 more than 5. |  | Emphasis should be on the language <br> ' 1 more than 5 is equal to 6.' <br> ' 2 more than 5 is 7. ' <br> ' 8 is 3 more than 5.' |



## Focus: Adding with 2 digit numbers.

Children should explore and understand how to use blank numberlines to add using their knowledge of place value and how to partition numbers in different ways. Once confident they should move onto written partitioning methods.


## Key Vocabulary

Add, more, plus, and, make, altogether, total, equal to, equals, the same as, double, most, count on, numberline, sum, tens, ones, partition, addition, column, tens boundary

## Key Skills

- Add a 2 digit number and units and a 2 digit number and 10 s.
- Add pairs of 2 digit numbers.
- Add three signle digit number.
- Know and show that adding can be done in any order (the commutative law).
- Recall bonds to 20 and multiple of 10 bonds to 100.
- Count in steps of 2,3 and 5 and count in 10 s from any number.
- Understand the place value of 2 -digit numbers (tens and ones).
- Compare and order numbers to 100 using < > and = signs.
- Read and write numbers to at least 100 in numerals and words.
- Solve contextual addition problems.

| Objective \& Strategy | Concrete | Pictorial | Abstract |
| :---: | :---: | :---: | :---: |
| Adding multiples of ten | $50=30=20$ <br> Model using dienes and bead strings | $\begin{gathered} 3 \text { tens }+5 \text { tens }=\ldots \text { tens } \\ 30+50= \end{gathered}$ <br> Use representations for base ten. | $\begin{aligned} & 20+30=50 \\ & 70=50+20 \\ & 40+\square=60 \end{aligned}$ |
| Use known number facts <br> Part part whole | Children explore ways of making numbers within 20 | $\begin{gathered} \square+\square=20 \\ 20-\square=\square \\ \square+\square=20 \\ \square=\square \\ \end{gathered}$ | $\square$ $+1=16$ $16-1=$ $\square$ $1+$ $\square$ $]=16$ <br> 16 - $\square$ $=1$ |
| Using known facts | $\begin{aligned} & \square_{\square} \square+\square_{\square} \quad=\quad \square_{\square} \square \square_{\square}{ }_{\square} \\ & \square \square \square \square \square \square \square \square \square \square \square \end{aligned}$ | Children draw representations of $\mathrm{H}, \mathrm{T}$ and O | $3+4=7$ <br> leads to $30+40=70$ <br> leads to $300+400=700$ |
| Bar model | $3+4=7$ | $7+3=10$ | 23 25 <br> $?$ $23+25=48$ |


| Objective \& Strategy | Concrete | Pictorial | Abstract |  |
| :---: | :---: | :---: | :---: | :---: |
| Add a two digit number and ones | $17+5=22$ <br> Use ten frame to make 'magic ten <br> Children explore the pattern. $\begin{aligned} & 17+5=22 \\ & 27+5=32 \end{aligned}$ | Use part part whole and number line to model. | $17+5=22$ <br> Explore related facts $17+5=22$ <br> $5+17=22$ <br> $22-17=5$ $22-5=17$ |  |
| Add a 2 digit number and tens | $25+10=35$ <br> Explore that the ones digit does not change |  | $\begin{aligned} & 27+10=37 \\ & 27+20=47 \\ & 27+\square=57 \end{aligned}$ |  |
| Add two 2-digit numbers | ARARAB <br> Model using dienes, place value counters and numicon | Use number line and bridge ten using part whole if necessary. |  $\begin{gathered} 20+40=60 \\ 5+7=12 \\ 60+12=72 \end{gathered}$ |  |
| Add three 1-digit numbers | Combine to make 10 first if possible, or bridge 10 then add third digit | $\int_{a^{4}}^{8}+b_{0}^{4} \infty_{0}^{8}+b_{0}^{8}$ <br> Regroup and draw representation. | $\begin{aligned} 4+7+6 & =10+7 \\ & =17 \end{aligned}$ <br> Combine the two numbers that make/ bridge ten then add on the third. |  |

## Addition

## Focus: Adding with numbers up to 3 digits

In year 3 we will move to the traditional column method and to support this, children will first apply their partitioning skills to the partitioning column method.


## Key Vocabulary

Add, more, plus, and, make, altogether, total, equal to, equals, the same as, double, most, count on, numberline, sum, tens, units, partition, addition, column, tens boundary, hundreds boundary,
increase, vertical, exchange, expanded, compact

## Key Skills

- Read and write numbers to 1000 in numerals and words.
- Add 2 digit number mentally including those that bridge 100.
- Add a 3 digit number and ones, a 3 digit number and 10 s and a 3 digit number and 100 s mentally.
- Estimate answers to calculations, using the inverse operation to check.
- Solve problems, including missing number problems using number facts and place value.
- Recognise the place value of each digit in a 3 digit number (hundreds, tens and ones).
- Continue to practice many different mental addition strategies including adding to the nearest multiple of 10, 100, 1000 and adjusting, using number bonds, using near doubles, partitioning and recombining etc.



## Mear Three Addition

## Addition

## Focus: Adding with numbers up to 4 digits

In year 4 children will consolidate their use of the traditional column method and will be able to use it confidently to add numbers up to 4 digits. This could include exchanging ones, tens and hundreds.


## Remember!

1) The ones must be added first!
2) 'Exchange' numbers underneath the bottom line!
3) Reinforce the place value! It is not 6 add 8 , it is 6 tens add 8 tens!

## Key Vocabulary

Add, more, plus, and, make, altogether, total, equal to, equals, the same as, double, most, count on, numberline, sum, tens, ones, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, carry, expanded, compact, thousands, digits, inverse.

## Key Skills

- Select most appropriate method: mental, jottings or written and explain why.
- Recognise the place value of every digit in a 4 digit number.
- Round any number to the nearest 10,100 or 1000.
- Estimate and use inverse operations to check answers.
- Solve 2 step problems in different contexts, picking the correct operation to use.
- Find 100 more or less than a number.
- Continue to use a wide range of mental addition methods.
- Add numbers with up to 4 digits using column addition.


## Addition

## Focus: Adding with more than 4 digits

In year 5 children will now use the column method to add decimal numbers in the context of money and measures. It is important that children have place value skills beyond 4 digits here and fully understand what a decimal number represents.


Children should be working with numbers greater than 4 digits including numbers in the ten thousands and hundred thousands.


Children need to start using the column method to add more than two values, still considering place value very carefully.

## Remember!

1) It is important that children say 6 tenths add 7 tenths so they understand that they are adding part of a number not a whole number. 2) Empty places should be filled with a zero to show the value of that place.

## Key Vocabulary

Add, more, plus, and, make, altogether, total, equal to, equals, the same as, double, most, count on, numberline, sum, tens, ones, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, carry, expanded, compact, thousands, hundreds, digits, inverse, decimal place, decimal point, tenths, hundredths, thousandths.

## Key Skills

- Add increasingly large numbers mentally using an expanding range of strategies.
- Use rounding to check answers and make estimates.
- Understand the place value of tenths and hundredths.
- Solve multi step problems in different contexts, deciding which operations and methods to use and explaining why.
- Read, write, order and compare number to 1 million.
- Round any number to 1 million to the nearest $10,100,1000,10000$ or 100000.
- Add numbers with more than 4 digits using column addition.


## Addition

## Focus: Adding several numbers with an increasing level of complexity

In year 6 children need to use all the previous adding skills developed to add several numbers with a variety of different decimal places. Many of these problems will be in the context of money or measures.


Children need to use their knowledge of the decimal point to line up their amounts correctly in the column. Zeroes should be added to support place value, showing that there is no value to add.


## Key Voabulary

Add, more, plus, and, make, altogether, total, equal to, equals, the same as, double, most, count on, numberline, sum, tens, ones, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, carry, expanded, compact, thousands, hundreds, digits, inverse, decimal place, decimal point, tenths, hundredths, thousandths, integer

## Key Skills

- Solve problems mentally, including those with mixed operations and large numbers, using all the mental strategies learnt in previous years.
- Solve multi step problems in context, deciding which operations and methods to use,
- Use estimation to check answers to a calculation.
- Read, write order and compare numbers to 10 million and understand the value of each digit.
- Round any whole number to the nearest 10, 100, 1000, 10 000, 100000,1000000 or 10000000
- Round decimal numbers to the nearest whole number.

|  | Concrete | pitororial | Abstrat |
| :---: | :---: | :---: | :---: |
| cosme |  |  | $\begin{array}{r} 3517 \\ +\quad 396 \\ \hline 3913 \end{array}$ <br> Continue from previous work to carry Relate to money and measur |
|  |  |  |  |
| Y6-add several num- bers of increasing com- plexity Including adding money, measure and decimals with different numbers of decimal points. | ${ }_{4 \times 5}$ | $4{ }^{405}$ |  |

## Subtraction in EXIFS

Maths for young children should be meaningful. Where possible, concepts should be taught in the context of real life.

## GUIDANCE / MODELS AND IMAGES

Children begin with mostly pictorial representations or real contexts.
Concrete apparatus is used to relate subtraction to taking away and counting how many objects are left.

Concrete apparatus models the subtraction of 2 objects from a set of 5 .

Construct number sentences verbally or using cards to go with practical activities.


Children are encouraged to read sentences aloud in different ways "five subtract one leaves four" "four is equal to five subtract one" "four is the same as five subtract one"

Children make a record in pictures, words or symbols of subtraction activities.

Solve simple problems using fingers


Number tracks can be introduced to count back and to find one less: What is 1 less than 9 ? 1 less than 20?

Number lines can then be used alongside number tracks and practical apparatus to solve subtraction calculations and word problems. Children count back showing hops back on the number back.


Children will need opportunities to look at and talk about different models and images as they move between representations.

## KEY VOCABULARY

Games and songs can be useful way to begin using vocabulary involved in addition i.e.

Five little men in a flying saucer

$$
\begin{aligned}
& \text { take (away) } \\
& \text { estimate } \\
& \text { leave }
\end{aligned}
$$

how many are left / left over?
how many have gone/
one less, two less, ten less...?
how many fewer is ...?
than...?
difference between
the same as
hopping back

## Focus: Subtracting from numbers up to 20

In Year 1, children will use numberlines, objects and visual models to understand subtraction as taking away but also as the difference between or distance between two numbers.


Hundred squares, number tracks, counting objects and real life objects should all be used as well to explore subtraction in a variety of practical contexts.


> Mental subtraction is equally important in year 2 and children should practice recalling subtraction facts up to and within 10 and 20 . In year 1 children should also be taught about subtracting zero.

## Key Vocabulary

Equal to, take, take away, less, minus, subtract, leaves, distance between, difference between, how many more, how many fewer/less than, most, least, count back, how many left, how much less is_?

## Key Skills

- Given a number, say one more or one less.
- Count to and over 100, forward and back from any number in 1s.
- Represent and use subtraction facts to 20 and within 20.
- Subtract with one digit and 2 digit numbers to 20 , including zero.
- Solve one step problems that involve subtraction using objects, pictures and numbered lines.
- Read and write numbers to 100 in numerals.
- Write numbers in words to 20 s, including correct spelling.

| Objective \& Strategy | Concrete | Pictorial | Abstract |
| :---: | :---: | :---: | :---: |
| Taking away ones. | Use physical objects, counters , cubes etc to show how objects can be taken away. <br> $6-4=2$ <br> $4-2=2$ | $15-3=12$ <br> Cross out drawn objects to show what has been taken away. | $7-4=3$ $16-9=7$ |
| Counting back | Move objects away from the group, counting backwards. <br> Move the beads $\square$ along the bead string as you count 0epeesee on backwards. | Count back in ones using a number line. | Put 13 in your head, count back 4. What number are you at? |
| Find the <br> Difference | Compare objects and amounts <br> 'Seven is 3 more than four' 4 <br> 'I am 2 years older than my sister' <br> Lay objects to represent bar model. | Count on using a number line to find the difference. | Hannah has12 sweets and her sister has 5. How many more does Hannah have than her sister.? |


|  <br> Strategy | Concrete | Pictorial | Abstract |
| :---: | :---: | :---: | :---: |
| Represent and use number bonds and related subtraction facts within 20 <br> Part Part Whole model | Link to addition. Use PPW model to model the inverse. <br> If 10 is the whole and 6 is one of the arts, what $s$ the other part? $10-6=4$ | Use pictorial representations to show the part. | Move to using numbers within the part whole model. |
| Make 10 | Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5. | Jump back 3 first, then another 4 . Use ten as the stopping point. | $16-8$ <br> How many do we take off first to get to 10? How many left to take off? |
| Bar model | $5-2=3$ |  | 8 2$\begin{aligned} & 10=8+2 \\ & 10=2+8 \\ & 10-2=8 \\ & 10-8=2 \end{aligned}$ |

## Subtraction

## Focus: Subtracting with 2 digit numbers

 In year 2 children will start to use blank numberlines to subtract by counting back which will greatly support the development of mental subtraction skills. Base 10 is also a super subtraction tool and should be used alongside blank numberline methods.

For $47-23=24$, children should start by partitioning the tens number and subtracting that first by counting back in tens. They will then subtract the units number and subtract that by counting back in 1 s .

Once confident with efficient jumps, children are ready to subtract by bridging through 10 , again partitioning is very important here and the children will need to be very confident with partitioning in different ways.


Counting on as a mental method
Counting on is a super mental method! It is especially useful for finding the difference problems and numbers that are close together. It is important that children understand that although they are counting on, they are finding the difference which is subtraction!


## Key Vocabulary

Equal to, take, take away, less, minus, subtract, leaves, distance between, difference between, how many more, how many fewer/less than, most, least, count back, how many left, how much less is_?, count on, strategy, partition, tens, ones

## Key Skills

- Recognise the place value of each digit in a 2 digit number.
- Recall and use subtraction facts to 20 fluently, use to derive related facts to 100.
- Subtract using objects, images, 100 squares and mentally including a two digit number and ones, a two digit number and 10 s and two 2 digit numbers.
- Understand and show that subtraction calculations cannot be done in any order.
- Use the inverse relationship between + and - to check calculations and solve missing number problems.
- Solve simple subtraction problems in context using written and mental methods.
- Read and write numbers to at least 100 in numerals and words.

| Objective \& Strategy | Concrete | Pictorial | Abstract |
| :---: | :---: | :---: | :---: |
| Regroup a ten into ten ones | Use a PV chart to show how to change a ten into ten ones, use the term 'take and make' | $\begin{aligned} & \sum_{3}^{3} \sum_{3}^{3} \quad \\ & 20-4= \end{aligned}$ | $20-4=16$ |
| Partitioning to subtract without regrouping. <br> 'Friendly numbers' | $34-13=21$ <br> Use Dienes to show how to partition the number when subtracting without regrouping. | Children draw representations of Dienes and cross off. $43-21=22$ | $43-21=22$ |
| Make ten strategy <br> Progression should be crossing one ten, crossing more than one ten, crossing the hundreds. | $34-28$ <br> Use a bead bar or bead strings to model counting to next ten and the rest. | Use a number line to count on to next ten and then the rest. | $93-76=17$ |
|  |  |  |  |

## Subtraction

## Focus: Subtracting with $\mathbf{2}$ and $\mathbf{3}$ digit numbers

Children will consolidate their knowledge of counting back and counting on using a blank numberline to subtract. They will use these methods both written and mentally. Once children become fully confident they will be ready to move on to the partitioning column method of subtraction.


## Key Vocabulary

E uals, take, take away, less, minus, subtract, leaves, distance between, difference between, how many more, how many fewer/less than, most, least, count back, how many left, how much less is_?, count on, strategy, partition, tens, ones, exchange, decrease, hundreds, value, digit

## Key Skills

- Subtract mentally: a 3 digit number and 1 s , a 3 digit number and 10 s and a 3 digit number and 100 s.
- Estimate answers and use the inverse to check.
- Solve problems in different contexts, including missing number problems.
- Find 10 or 100 more or less than a given number.
- Recognise the place value in a 3 digit number, $100 \mathrm{~s}, 10 \mathrm{~s}$ and 1 s .
- Solving finding the difference problems using counting on.
- Reading and writing numbers up to 1000 in numerals and words.
- Practise and develop mental strategies including subtracting near multiples of 10 and adjusting, counting on etc.

| Objective \＆ Strategy | Concrete | Pictorial | Abstract |
| :---: | :---: | :---: | :---: |
| Column subtraction without regrouping （friendly numbers） | Use base 10 or Numicon to model |  <br> Darw representations to support under－ standing | $\begin{gathered} 47-24=23 \\ -40+7 \\ -20+4 \\ \hline 20+3 \\ \hline \end{gathered}$ <br> Intermediate step may be needed to lead to clear subtraction under－ standing． |
| Column subtraction with regrouping | Begin with base 10 or Numicon．Move to pv counters，modelling the exchange of a ten into tten ones．Use the phrase＇take and make＇for exchange． | Children may draw base ten or PV counters and cross off． | Begin by parti－ tioning into pv columns $\begin{gathered} 728-582=146 \\ { }^{H} \quad 10 \\ { }^{\prime} 7 \\ \hline \end{gathered}$ <br> Then move to formal method． |
|  |  |  |  |



## Key Vocabulary

Equal to, take, take away, less, minus, subtract, leaves, distance between, difference between, how many more, how many fewer/less than, most, least, count back, how many left, how much less is_?, count on, strategy, partition, tens, ones, exchange, decrease, hundreds, value, digit, inverse.

## Key Skills

- Subtract by counting on where numbers are close together or they are near to multiples of 10,100 etc.
- Children select a mental, written or jotting method depending on what the problem requires.
- Children estimate and use the inverse operation to check a problem.
- Children solve 2 step problems involving + and -, picking the correct operation and method.
- Children solve simple money and measure problems with fractions and decimals.
- Find 1000 more or 1000 less than a given number.
- Count backwards through zero including negative numbers.
- Recognise the place value of each digit in a 4 digit number.
- Round any number to the nearest 10,100 or 1000.
- Solve number and practical problems that involve increasingly large positive integers.


## Subrpaction

## Focus: Subtracting with numbers beyond 4 digits including decimals

 Children in year 5 will continue to use the compact column method of subtraction to solve problems including those where exchanging is required. They will subtract larger integers and begin to subtract decimal amounts.

## Key Vocabulary

Equal to, take, take away, less, minus, subtract, leaves, distance between, difference between, how many more, how many fewer/less than, most, least, count back, how many left, how much less is_?, count on, strategy, partition, tens, ones, exchange, decrease, hundreds, value, digit, inverse, tenths, hundredths, decimal place, decimal

## Key Skills

- Subtract mentally with increasingly large numbers.
- Use rounding and estimation to check answers to calculations.
- Solve addition and subtraction multi step problems, deciding which operations to use and why.
- Read, write, order and compare numbers to at least 1 million and understand the value of each digit.
- Count forwards or backwards in steps of powers of 10 up to 1 million.
- Understand negative numbers in context and count forwards and backwards through 0.
- Round any number up to 1 million to the nearest $10,100,1000,10000$ and 100000.


## Subtraction

Focus: Subtracting with increasingly complex numbers including decimals
In year 6, children need to use mental methods and the compact column method of subtraction to solve an increasingly complex range of calculation including those with integers, those with decimals and those with mixed numbers.


## Key Vocabulary

Equal to, take, take away, less, minus, subtract, leaves, distance between, difference between, how many more, how many fewer/less than, most, least, count back, how many left, how much less is_?, count on, strategy, partition, tens, ones, exchange, decrease, hundreds, value, digit, inverse, tenths, hundredths, decimal place, decimal

## Key Skills

- Solve addition and subtraction multi step problems in context, deciding which operations to use and why.
- Read, write, order and compare numbers to at least 10 million and understand the value of each digit.
- Round any whole number up to 10 million to the nearest $10,100,1000,10000,100000$, or 1 million.
- Use negative numbers in context and calculate intervals across zero.
- Look at a calculation and decide whether you need to use a mental method, a jotting, a written method or a calculator to solve.

|  <br> Strategy | Concrete | Pictorial | Abstract |
| :---: | :---: | :---: | :---: |
| Subtracting tens and ones Year 4 subtract with up to 4 digits. <br> Introduce decimal subtraction through context of money | $234-179$  <br> Model process of exchange using Numicon, base ten and then move to PV counters. | Children to draw pv counters and show their exchange-see Y3 | Use the phrase 'take and make' for exchange |
| Year 5-Subtract with at least 4 digits, including money and measures. <br> Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal | As Year 4 | Children to draw pv counters and show their exchange-see Y3 | $\begin{array}{r} { }^{2} 8^{10} x^{109}{ }^{\circ} 6 \\ -\quad 2128 \\ \hline 28,928 \end{array}$ <br> Use zeros for placeholders. $\begin{array}{r} 6{ }^{10} x^{\prime} 6^{8} 9 \cdot 0 \\ -\quad 372 \cdot 5 \\ \hline 6796.5 \end{array}$ |
| Year 6-Subtract with increasingly large and more complex numbers and decimal values. |  |  |  |

## Multiplication in EVIF

Maths for young children should be meaningful. Where possible, concepts should be taught in the context of real life.

## GUIDANCE / MODELS AND IMAGES

The link between addition and multiplication can be introduced through doubling.
If available, numicon is used to visualise the repeated adding of the same number. These can be drawn around or printed as a way of recording.

Children being with mostly pictorial representations:


How many groups of 2 are there? $2+2+2+2+2$, so 5 groups of 2
Real life contexts and use of practical equipment to count in repeated groups of the same size:


How many wheels are there altogether?


How much money do I have?

Count in twos, fives, tens both aloud and with objects.
 objects

Children are given multiplication problems set in a real life context. Child are encouraged to visualise the problem.

How many fingers on two hands? How many sides on three triangles? How many legs on four ducks?


Children are encouraged to read number sentences aloud in different ways "five times two makes ten" "ten is equal to five multiplied by two" "ten is the same as five lots of two"

## KEY VOCABULARY

Lots of
Groups of
Times
Multiply
Multiplied by
Multiple of
Once, twice, three times... ten times...
... times as (big, long, wide... and so on) repeated addition
double
estimate add again and again

Focus: Repeated addition with objects, arrays and pictorial representations.
In year one children will be exposed to many different multiplication based activities in a variety of contexts. Much of this will be repeated addition activities or be linked to counting in $2 s, 5$ s or 10 s .


Children use images and pictorial representations to solve simple problems that involve repeated addition. They may wish to use the picture to support or use other equipment. Adult support at this stage is to be expected.

Some children may start to see the link between the problem below and counting in 5 s and be able to use mental skills to solve the problem.

There are 5 roses in each garden. How many roses in 3 gardens?

$5+5+5=15$

## Key Vocabulary <br> Groups of, lots of, times, array, altogether, multiply, count

## Key Skills

- Count in multiples of 2,5 and 10.
- Solve 1 step problems involving multiplication using objects, arrays or pictures with support.
- Make connections between arrays and counting in $2 \mathrm{~s}, 5 \mathrm{~s}$ and 10 s .
- Begin to understand doubling using objects and pictorial representations.
- Solve practical problem solving activities counting equal sets or groups.
- Have lots of practice counting and bundling groups of objects into $2 \mathrm{~s}, 5 \mathrm{~s}$ and 10 s.

| Objective \& Strategy | Concrete | Pictorial | Abstract |
| :---: | :---: | :---: | :---: |
| Doubling | Use practical activities using manipultives including cubes and Numicon to demonstrate doubling | Draw pictures to show how to double numbers <br> Double 4 is 8 $\square$ $\square$ $\square$ $\square$ $\square$ $\square$ $\square$ $\square$ | Partition a number and then double each part before recombining it back together. |
| Counting in multiples | Count the groups as children are skip counting, children may use their fingers as they are skip counting. | Children make representations to show counting in multiples. | Count in multiples of a number aloud. Write sequences with multiples of numbers. $2,4,6,8,10$ $5,10,15,20,25,30$ |
| Making equal groups and counting the total | Use manipulatives to create equal groups. | Draw to show $2 \times 3=6$ <br> Draw and make representations | $2 \times 4=8$ |


| Objective \& Strategy | Concrete | Pictorial | Abstract |
| :---: | :---: | :---: | :---: |
| Repeated addition | Use different objects to add equal groups | Use pictorial including number lines to solve prob There are 3 sweets in one bag. <br> How many sweets are in 5 bags altogether? | Write addition sentences to describe objects and pictures. |
| Understanding arrays | Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc. | Draw representations of arrays to show understandino | $\begin{gathered} 3 \times 2=6 \\ 2 \times 5=10 \end{gathered}$ |
|  |  |  |  |

Focus: Multiplying using arrays and repeated addition- 2,3,4,5,10x table facts
In year 2 children will be aware of simple arrays and pictorial representations and understand what they mean. In year 2 children will develop the knowledge of how to make their own arrays to solve a problem and also how repeated addition on a numberline can get them to a solution.


## Key Vocabulary

Groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times

## Key Skills

- Count in steps of 2,3 and 5 from zero and in 10 s from any number.
- Recall and use multiplication and division facts for the 2,5 AND 10 times tables.
- Recognise odd and even numbers.
- Write and calculate number statements using the $x$ and $=$ signs.
- Show that multiplication can be done in any order (the commutative law).
- Solve a range of multiplication problems using objects, arrays, repeated addition, mental methods and multiplication facts.
- Use and become familiar with all of the above multiplication language.

| Objective \& Strategy | Concrete | Pictorial | Abstract |
| :---: | :---: | :---: | :---: |
| Doubling | Model doubling using dienes and PV counters. | Draw pictures and representations to show how to double numbers | Partition a number and then double each part before recombining it back together. |
| Counting in multiples of 2, 3, 4, 5, 10 from 0 <br> (repeated addition) | Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models. $5+5+5+5+5+5+5+5=40$ | Number lines, counting sticks and bar models should be used to show representation of counting in multiples. <br> 3 <br> 3 <br> 3 <br> 3 | Count in multiples of a number aloud. <br> Write sequences with multiples of numbers. $\begin{aligned} & 0,2,4,6,8,10 \\ & 0,3,6,9,12,15 \\ & 0,5,10,15,20,25,30 \end{aligned}$ $4 \times 3=$ |



## Focus: Multiplying $\mathbf{2}$ digit numbers by $\mathbf{1}$ digit numbers

In year 3 children will move on from arrays and start using the grid method of multiplication. It is essential that before children move onto the grid method they are completely confident with all previous methods and have a solid grounding with mental methods and partitioning.


## Key Vocabulary

Groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, ${ }^{\text {co }}$ mmutative, sets of, equal groups, times as big as, once, twice, three times, partition, grid method, multiple, product, tens, ones, value

## Key Skills

- Recall and use multiplication facts for the $2,3,4,5,6$ and 10 multiplication tables and multiply multiples of 10.
- Write and calculate number sentences using known x tables.
- Answer 2 digit $\times 1$ digit problems using mental and written methods.
- Solve multiplication problems in context including missing number problems.
- Develop mental strategies using commutativity (e.g. $4 \times 12 \times 5=4 \times 5 \times 12=20 \times 12=240$ ) and for missing number problems.

| Objective \& Strategy | Concrete | Pictorial | Abstract |
| :---: | :---: | :---: | :---: |
| Grid method | Show the links with arrays to first introduce the grid method. <br> 4 rows of 10 4 rows of 3 <br> Move onto base ten to move towards a more compact method. 4 rows of 13 <br> Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows <br> Fill each row with 126 <br> Add up each column, starting with the ones making any exchanges needed <br> Then you have your answer. | Children can represent their work with place value counters in a way that they understand. <br> They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below. <br> Bar model are used to explore missing numbers $4 \times \square=20$ | Start with multiplying by one digit numbers and showing the clear addition alongside the grid. $210+35=245$ <br> Moving forward, multiply by a 2 digit number showing the different rows within the grid method. |

## Focus: Multiplying 2 and $\mathbf{3}$ digit numbers by 1 digit numbers

In year 4 children need to use the grid method confidently to solve problems where a 2 or 3 digit number is multiplied by a one digit number. They need to move on to the use of short multiplication to solve 3 digit number multiplied by 1 digit problems.


Add up 3000, 50 and 15 to make 3065.

The grid method is extended in year 4 so children will now multiply 3 digit numbers by 1 digit numbers. When adding the 3 answers up to create a total, column addition could be used to ensure accuracy, especially where bridging will be needed.

## $613 \times 5=3065$

It is at this stage that approximation and estimation should become a regular part of classroom practice. Children should approximate an answer before using a method so they know if there answer is accurate or not.
$253 \times 9$ is approximately $250 \times 10=2500$

## Key Vocabulary

Groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times, partition, grid method, multiple, product, tens, ones, value, inverse

## Key Skills

- Count in multiples of $6,7,8,9,25$ and 1000 .
- Recall multiplication facts for all multiplication tables up to $12 \times 12$.
- Recognise place value of digits in up to 4 digit numbers.
- Multiply large numbers and multiple values mentally using place value, known facts and derived facts.
- Use commutativity mentally to solve problems.
- Solve problems in a range of contexts that are increasingly complex.

| Objective \& Strategy | Concrete | Pictorial | Abstract |
| :---: | :---: | :---: | :---: |
| Grid method recap from year 3 for 2 digits x 1 digit <br> Move to multiplying 3 digit numbers by 1 digit. (year 4 expectation) | Use place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows <br> \| Fill each row with 126 <br> Add up each colu les making any exchanges needed | Children can represent their work with place value counters in a way that they understand. <br> They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below. | Start with multiplying by one digit numbers and showing the clear addition alongside the grid. $210+35=245$ |
| Column multiplication | Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. $321 \times 2=642$ <br> It is important at this stage that they always multiply the ones first. <br> The corresponding long multiplication is modelled alongside | $x$ 300 20 7 <br> 4 1200 80 28 <br> The grid method my be used to show how this relates to a formal written method. <br> Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods. |  |

Focus: Multiplying up to $\mathbf{4}$ digits by 1 or 2 digits
In year 5 children will continue to use short multiplication to solve increasingly richer problems that involve multiplying by 1 digit. They will then move on to long multiplication for problems that involve multiplying by 2 digits. Approximation will play an important part- with children making approximations before using long multiplication to help check their answer is correct.


## Key Vocabulary

Groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, e ual groups, times as big as, once, twice, three times, partition, grid method, multiple, product, tens, ones, value, inverse , square, factor, integer, decimal, short/long multiplication, exchange

## Key Skills

- Identify multiples and factors, using secure x table facts to $12 \times 12$.
- Solve problems where larger numbers are decomposed into their factors.
- Multiply and divide integers and decimals by 10,100 and 1000.
- Recognise and use square and cube numbers and their notation.
- Solve problems that have different combinations of operations, picking the most useful methods.

Focus: Consolidating short and long multiplication, multiplying decimals by 1 digit In year 6 children will consolidate all they know about short and long multiplication before they go to Secondary school. They will also learn the new skill of using short multiplication to multiply decimal numbers to 2 decimal places.


## Key Vocabulary

Groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times, partition, grid method, multiple, product, tens, ones, value, inverse, square, factor, integer, decimal, short/long multiplication, carry, tenths, hundredths, decimals

## Key Skills

- Multiply up to 4 digits by 2 digits using long multiplication.
- Solve mixed operation and large number problems using mental methods.
- Solve multi step problems involving a range of operations.
- Estimate and approximate answers of problems to improve accuracy.
- Round any integer to the determined level of accuracy.

| Objective \& Strategy | Concrete | Pictorial | Abstract |
| :---: | :---: | :---: | :---: |
| Column Multiplication for 3 and 4 digits $\times 1$ digit. |  <br> It is important at this stage that they always multiply the ones first. <br> Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. $321 \times 2=642$ | $x$ 300 20 7 <br> 4 1200 80 28 |  |
| Column multiplication | Manipulatives may still be used with the corresponding long multiplication modelled alongside. | Continue to use bar modelling to support problem solving |  1 8  <br> $\times$ 1 3  <br>  5 4  <br>  2   <br> 1 8 0  <br> 2 3 4  <br> $18 \times 3$ on the first row <br> ( $8 \times 3=24$, carrying the 2 for 20 , then $1 \times 3$ ) <br> $18 \times 10$ on the 2nd row. Show multiplying by 10 by putting zero in units first |



## Division in $\mathbb{E} X \mathbb{E}$

Maths for young children should be meaningful. Where possible, concepts should be taught in the context of real life.

## GUIDANCE / MODELS AND IMAGES

The ELG states that children solve problems including doubling, halving and sharing.
Children need to see and hear representations of division as both grouping and sharing.

Division can be introduced through halving.
Children begin with mostly pictorial representations linked to real life contexts.


Mum has 6 socks. She grouped them into pairs - how many pairs did she make? How many socks did she have altogether?


Although not explicit in the development matters document, the sharing model is a useful way of introducing young children to fractions and calculating with fractions.

Setting the problems in a real life context and solving them with concrete apparatus will support children's understanding.
"I have got a whole pizza to share between two people. Can you cut the pizza in half?"
Children make a record in pictures, words or symbols of division activities.

Children are encouraged to have a go at recording the calculation which has been carried out.

## KEY VOCABULARY

halve
share
share, share equally
one each, two each, three each...
groups in pairs, threes...
tens
equal groups of
divide
divided by
divided into
left, left over
estimate
fraction
half
halves
whole
quarter
$\square$

## Division

## Focus: Grouping and sharing small quantities without remainders

As an introduction to division, children in year 1 will solve problems in familiar and relevant contexts where they have to group and share. They will use objects and pictorial representations to solve problems and they will begin to use counting in $2 \mathrm{~s}, 5 \mathrm{~s}$ and 10 s to support their problems solving.

A farmer has 15 roses and shares them between 3 friends. How many roses do they each get?


15 roses shared between $3=5$ roses each


Bats fly in groups of 2 . How many groups of 2 will there be if there are 8 bats?


8 bats shared into groups of $2=2$ bats in each group

## Key Vocabulary

Share, share equally, one each, two each..., group, groups of, lots of, array

## Key Skills

- Solve one step problems involving multiplication and division using concrete objects with support from adults.
- Children use grouping and sharing to understand division and to begin to understand finding simple fractions.
- Children make connections between arrays and counting in $2 \mathrm{~s}, 5 \mathrm{~s}$ and 10 s .
- Children use halving and understand that this is the same as sharing into 2 equal groups.



## Division

## Focus: Grouping and sharing larger quantities using written methods and symbols

Children will continue to use the methods of sharing and grouping in division with objects to support their understanding of arrays for sharing and grouping and the division numberline for grouping.

To solve problems such as $15 \div 3=$, children will share 15 objects into 3 groups like in the first array or make groups of 3 until they get to 15 , like in the second image.


Completing both of these processes will help children see the link between sharing and grouping but also the link between $15 \div 3=5$ and $15 \div 5=3$.
Children will start to group on a numberline- which will help
cement their understanding of division as grouping. When
grouping on a numberline, children will start with a zero at the
beginning and will write the dividend at the end of the line, they
will then jump in steps of the divisor. The example to the right
shows a numberline for the calculation $12 \div 3=4$ as there were
4 jumps of 3 to get to 12 .

## Key Vocabulary

Share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over

## Key Skills

- Count in steps of 2,3 and 5 from 0 .
- Recall and use $x$ and $\div$ facts for the 2,5 and 10 times tables.
- Solve division problems and write division number sentences for problems.
- Understand that division is not commutative unlike multiplication.
- Solve increasingly challenging division problems using concrete objects, arrays, and simple written methods such as grouping on a numberline.

| Objective \& Strategy | Concrete | Pictorial | Abstract |
| :---: | :---: | :---: | :---: |
| Division as sharing | I have 10 cubes, can you share them equally in 2 groups? | Children use pictures or shapes to share quantities. <br> Children use bar modelling to show and support understanding. <br> $12 \div 4=3$ | $12 \div 3=4$ |
| Division as grouping | Divide quantities into equal groups. <br> Use cubes, counters, objects or place value counters to aid understanding. | Use number lines for grouping <br> $12 \div 3=4$ <br> Think of tue dar as a wrivie. split it mio the number of groups you are dividing by and work out how many would be within each group. $\begin{aligned} & 20 \div 5=? \\ & 5 \times ?=20 \end{aligned}$ | $28 \div 7=4$ <br> Divide 28 into 7 groups. How many are in each group? |

## Focus: Dividing 2 digit numbers by 1 digit numbers moving from numberline methods to

 short divisionChildren in year 3 will continue to use a numberline to solve division problems and will begin to jump more than one step at a time in the style of 'chunking'. Once confident they will move on to short division without any remainders.


Once confident children will begin to solve problems on a grouping numberline involving bigger numbers. To solve this effectively they will need to subtract chunks of the divisor. As you can see in the image for $92 \div 4$, a step of 10 groups of 4 has been jumped, followed by another step of 10 jumps, finally followed by a step of 3 jumps of 4 . This means that in total 4 was jumped 23 times


Once children are confident with numberline methods then they should start work on short division. First of all arrays should be used be used to show a division calculation, the same calculation should then be shown in the short multiplication method. Place value should be regularly discussed so children realize that they are partitioning the dividend and dividing the units then then tens by the divisor.

## Key Vocabulary

Share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, exchange remainder, multiple

## Key Skills

- Recall and use and $\div$ facts for the $2,3,4,5,6,8$ and $10 x$ tables (using doubling to connect the 2,4 and 8 x tables)
- Solving division problems where a 2 digit number is divided by a 1 digit number using mental and written.
- Solve problems in a variety of contexts including missing number problems.
- Pupils begin to derive related facts e.g. $9 \div 3=3$ means $90 \div 3=30$ or $90 \div 30=3$.
- Pupils develop confidence in written methods, moving from numberlines to short division.

| Objective \& Strategy | Concrete | Pictorial | Abstract |
| :---: | :---: | :---: | :---: |
| Division as grouping | Use cubes, counters, objects or place value counters to aid understanding. <br> 24 divided into groups of $6=4$ $96 \div 3=32$ | Continue to use bar modelling to aid solving division problems. $\begin{aligned} & 20 \div 5=? \\ & 5 \times ?=20 \end{aligned}$ | How many groups of 6 in $\begin{gathered} 24 ? \\ 24 \div 6=4 \end{gathered}$ |
| Division with arrays | Link division to multiplication by creating an array and thinking about the number sentences that can be created. $\begin{array}{rl} \operatorname{Eg} 15 \div 3=5 & 5 \times 3=15 \\ 15 \div 5=3 & 3 \times 5=15 \end{array}$ | Draw an array and use lines to split the array into groups to make multiplication and division sentences | Find the inverse of multiplication and division sentences by creating eight linking number sentences. $\begin{aligned} & 7 \times 4=28 \\ & 4 \times 7=28 \\ & 28 \div 7=4 \\ & 28 \div 4=7 \end{aligned}$ $28=7 \times 4$ $28=4 \times 7$ $4=28 \div 7$ $7=28 \div 4$ |


| Objective \& Strategy | Concrete | Pictorial | Abstract |
| :---: | :---: | :---: | :---: |
| Division with remainders. | $14 \div 3=$ <br> Divide objects between groups and see how much is left over <br> Example without $40 \div 5$ <br> Ask "How many <br> Example with re $38 \div 6$ | Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder. <br> Draw dots and group them to divide an amount and clearly show a remainder. <br> Use bar models to show division with remainders. <br> remainder: <br> $5 s$ in 40?" <br> mainder: <br> s, when it becomes inefficient to count in single m orded using known facts. | Complete written divisions and show the remainder using r . <br> ves <br> a remainder of 2 <br> ultiples, bigger |

## 

For larger numbers, when it becomes inefficient to count in single multiples, bigger jumps can be recorded using known facts.

## Division

## Focus: Consolidating and extending use of short division

Children in year 4 will continue to use short division to solve division problems. They will begin to work on remainders, including problems where there are remainders in the first numbers but not in the final answer.


## Key Vocabulary

Share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, carry, remainder, multiple, divisible by, factor

## Key Skills

- Recall multiplication and division facts for all numbers to $12 \times 12$.
- Use place value and known facts to derive facts mentally-including multiplying and dividing by 100,10 and 1.
- Practise mental methods and extend this to three digit numbers using derived facts- e.g. $100 \div 5=20$ so 20 $\times 5=100$.
- Solve two step problems with increasingly harder numbers in a range of contexts, using language to identify the correct operation.
- Correspondence problems should be introduced such as 3 cakes are shared equally between 10 children, 1 man has 6 cats so how many cats do 3 men have etc.


## Division

## Focus: Extending use of short multiplication to 4 digits and remainders

Children in year 5 will use short division to solve problems up to 4 digits long. For the first time they will use short division to solve problems that have a remainder in the final answer.


## Key Vocabulary

Share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, carry, remainder, multiple, divisible by, factor, quotient, prime number, prime factors, composite number (non-prime)

## Key Skills

- Multiply and divide numbers mentally, using known facts.
- Identify multiples and factors, including all factor pairs of a number and common factors between 2 numbers.
- Solve $x$ and $\div$ problems where larger numbers are decomposed into their factors.
- Multiply and divide whole numbers and decimals by 10, 100 and 1000.
- Use vocabulary of prime numbers, prime factors and composite numbers.
- Work out whether a number up to 100 is prime and know all prime numbers to 30 .
- Use and understand multiplication and division as inverses.
- Present division with remainders answers differently, showing the remainder as a fraction, decimal or whole number by rounding.
- Solve problems with a combination of all four operations including fraction scaling problems and problems involving simple rates.


Focus: Using short division to divide 4 digit numbers and express remainders as decimals and long division for dividing 2 digit numbers
In year 6, children will use short division to divide decimal numbers by single digit numbers. The final step of division will be long division which will be used to divide numbers by 2 digits.


The remainder in this answer would have been 1 but it has been expressed as a decimal. To do this, children need to insert a decimal point next to the units and carry the remainder over the decimal point. Zeroes are inserted to the right of the decimal point to show that there was no value.



$$
\square
$$

The focus in year 6 is not so much the method of short division but how the remainders are expressed- children need to express remainders as decimals and fractions- depending on the context of the question.

## T



To divide by 2 digit numbers, the children will use the method of long division. The example to the right clearly shows the method in the 'Burger' steps, where as the example to the left shows what a completed method would look like. Any remainders would need to be expressed in a way that matched the context of the problem.

| Divide: | $3 \longdiv { 2 \leftarrow }$ | 3 goes into 7 <br> -2 times... <br> with some extra! |
| :---: | :---: | :---: |
| Multiply: | $\frac{\sqrt{2}}{3 \longdiv { 7 5 }}$ | $2 \times 3=6$ |
| Subtract: | $\begin{gathered} 2 \\ 3 \longdiv { 7 5 } \\ -\frac{6}{1} \end{gathered}$ |  |
| Bring Down: | $\begin{array}{r} 2 \\ 3 \longdiv { 7 5 } \\ -6 \downarrow \\ \hline 15 \end{array}$ |  |
| Repeat: | $\begin{gathered} 25 \\ 3 \longdiv { 7 5 } \\ -\frac{6}{15} \\ \frac{-15}{0} \end{gathered}$ | $\begin{aligned} 15 \div 3 & =5 \\ 5 \times 3 & =15 \end{aligned}$ |

## Key Vocabulary

Share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, carry, remainder, multiple, divisible by, factor, quotient, prime number, prime factors, composite number (non-prime), common factor

## Key Skills

- Use multiplication and division facts up to $12 \times 12$ to solve more complex problems.
- Decide when to use short or long division and interpret remainders in a way that is appropriate to the problem.
- Perform mental calculations for problems involving large numbers and mixed calculations.
- Identify common factors, common multiples and prime numbers.
- Use estimation to check answers to calculations and determine accuracy.
- Use written methods of division to solve decimal problems up to 2 decimal places.
- Solve problems which require rounding to $10,100,1000$ and beyond.


## Long Division

Step 1-a remainder in the ones

| h to |
| :---: |
| 041 R 1 |
| 165 |

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).
4 goes into 16 four times.
4 goes into 5 once, leaving a remainder of 1 .
th ht o
$8 \longdiv { 0 4 0 0 \mathrm { R7 } }$

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds $(3,200)$.
8 goes into 32 four times $(3,200 \div 8=400)$
8 goes into 0 zero times (tens).
8 goes into 7 zero times, and leaves a remainder of 7 .

## Long Division

Step 1 continued...


When dividing the ones, 4 goes into 7 one time. Multiply $1 \times 4=4$, write that four under the 7 , and subract. This finds us the remainder of 3 .

Check: $4 \times 61+3=247$

> th hto 0402 $4 \longdiv { 1 6 0 9 }$ $\frac{-8}{1}$

When dividing the ones, 4 goes into 9 two times. Multiply $2 \times 4=8$, write that eight under the 9 , and subract. This finds us the remainder of 1 .

Check: $4 \times 402+1=1,609$


## Long Division

| Step 2-a remainder in any of the place values | 1. Divide. | 2. Multiply \& subtract. | 3. Drop down the next digit. |
| :---: | :---: | :---: | :---: |
|  | $\begin{gathered} { }^{n t o} \\ 2 \longdiv { 1 } \\ 2 \longdiv { 2 7 8 } \end{gathered}$ <br> Two goes into 2 one time, or 2 hundreds $\div 2=1$ hundred. | $\begin{gathered} \quad h t o \\ 1 \\ 2 \longdiv { 2 7 8 } \\ \frac{-2}{0} \end{gathered}$ <br> Multiply $1 \times 2=2$, write that 2 under the two, and subtract to find the remainder of zero. | $\begin{gathered} h t \circ \\ 18 \\ 2 \longdiv { 2 7 8 } \\ -\frac{2}{07} \end{gathered}$ <br> Next, drop down the 7 of the tens next to the zero. |
|  | Divide. | Multiply \& subtract. | Drop down the next digit. |
|  | $\begin{gathered} h+0 \\ 13 \\ 2 \longdiv { 2 7 8 } \\ -\frac{2}{07} \end{gathered}$ <br> Divide 2 into 7 . Place 3 into the quotient. | $\begin{gathered} h t o \\ 13 \\ 2 \longdiv { 2 7 8 } \\ \frac{-2}{0} 7 \\ -\quad 6 \\ \hline 1 \end{gathered}$ <br> Multiply $3 \times 2=6$, write that 6 under the 7 , and subtract to find the remainder of 1 ten. | $\begin{gathered} h t 0 \\ 13 \\ 2 \longdiv { 2 7 8 } \\ \frac{-2}{07} \\ -\quad 6 \\ \hline 18 \end{gathered}$ <br> Next, drop down the 8 of the ones next to the 1 leftover ten. |
|  | 1. Divide. | 2. Multiply \& subtract. | 3. Drop down the next digit. |
|  | $\begin{gathered} h t 0 \\ 139 \\ 2 \longdiv { 2 7 8 } \\ -\frac{2}{07} \\ -\quad 6 \\ \hline 18 \end{gathered}$ <br> Divide 2 into 18. Place 9 into the quotient. | $\begin{aligned} & h t o \\ & 139 \\ & 2 \longdiv { 2 7 8 } \\ & -\frac{2}{0} 7 \\ & -\quad 6 \\ & \hline 18 \\ & -18 \\ & \hline 0 \end{aligned}$ <br> Multiply $9 \times 2=18$, write that 18 under the 18 , and subtract to find the remainder of zero. | $\begin{gathered} h+0 \\ 139 \\ 2 \longdiv { 2 7 8 } \\ \frac{-2}{07} \\ -\quad 6 \\ \hline 18 \\ \frac{-18}{0} \end{gathered}$ <br> There are no more digits to drop down. The quotient is 139 . |

## Clossary of Terms

2-digit number- a number with 2 digits like $23,45,12$ or 60
3-digit number-a number with 3 digits like 123, 542, 903 or 561
Addition facts - knowing that $1+1=2$ and $1+3=4$ and $2+5=7$. Nomally we only talk about number facts with tota ls of 20 and under.
Array - An a rray is an a rrangement of a set of numbers or objects in rows and columns -it is mostly used to show how you can group objects for repeated addition or subtraction.
Bead String/Bar-a string with (usually 100) beads on, grouped by colour in tens. The bead string is a good bridge between a numbertrack and a numberline as it maintains the cardinality of the numbers whilst beginning to develop the concepts of counting 'spaces' rather than objects.
Bridging - when a calculation causes you to cross a 'ten boundary' or a 'hundred boundary' e.g. $85+18$ will bridge 100 .
Compact vertical - the name of the recommended written method for addition whereby the numbers are added in columns, 1 s first then 10s a nd so on. Where the total exceeds 10, the ten 1s are exchanged for a 10 and written below the answer line.
Concrete apparatus - objects to help children count and calculate- these are most often cubes (multilink) but can be anything they can hold and move including Cuisena ire rods, Dienes rods (hundreds, tens and units blocks), straws, Numic on, Place Value counters and much more.
Count all - when you add by counting all the items/objectse.g. to add 11 and 5 you would count out 11, then count out 5, then put them together and count them all to get 16 .
Count on - when you add (or sometimes subtract) by counting onwa rds from a given number. E.g. to add 11 and 5 you would count on 5 from 11 i.e. 12, 13, 14, 15, 16 Decimal number - a number with a decimal point e.g. 2.34 (said as two point three four)
Decomposition - the name of the recommended written method for subtraction whereby the smaller number is subtracted from the larger, 1s first then 10s a nd so on. Where the subtraction cannot be completed as the second number is larger than the first, a 10 is exchanged for ten 1s to facilitate this. This is the traditional 'borrowing' form of column method, which is different to the 'payback' method.
Dienes Rods (or Base 10) - this is a set of practic al equipment that represents the numbers to help children with place value and calculation. The Dienes rods show 1 s , $10 s, 100$ s and 1000 s as blocks of cubes that children can then combine. Dienes rods do not break up so the child hasto 'exchange' them for smaller or larger blocks where necessary.
Difference - the gap between numbers that is found by subtraction e.g. 7-5 can be read as' 7 take away 5' or as the 'difference between 7 and 5'
Dividend - the number being divided in a calculation
Divisor- the sma ller number in a division calc ulation.

Double - multiply a number by 2
Effic ient Methods - the method(s) that will solve the calculation most rapidly and easily
Equals - is worth the same as (be careful not to emphasise the use of = to show the a nswer)
Exchanging - Swapping a '10' forten ' $1 s^{\prime}$ or a ' 100 ' forten ' 10 s' or vice versa (used in addition and subtraction when 'moving' 'ten' or a 'hundred' from its column into the next column and splitting it up). Heavily relied upon for addition and subtraction of larger numbers. Skills in this can be built up practically with objects, then Dienes rods/base 10, then place value counters before relying on a solely written method.
Expanded Multiplication - a method for multiplic ation where each sta ge is written down and then added up at the end in a column
Factor- a number that dividesexactly into a nother number, without remainder Grid method - a method for multiplying two numbers together involving partitioning and multiplying each piece separately.
Grouping - an approach to division where the dividend is split into groups of the size of the divisor and the number of groups created are then counted.
Half - a number, shape or quantity divided into 2 equal parts
Halve - divide a number by 2
Integer-a whole number (i.e. one with no decimal point)
Inverse - the opposite operation. For example, addition is the inverse of subtraction and multiplication is the inverse of division.
Known Multiplication Facts - times ta bles and other number facts that can be recalled quickly to support with larger or related calculationse.g. if you know $4 \times 7$ then you also know $40 \times 70,4 \times 0.7$ etc.
Long Division - formal written of division where the remainders a re calculated in writing each time (extended version of short division)
Long Multiplication - formal written method of column multiplic ation
Multiple - a numberwhich is an exact product of a nother number i.e. a number which is in the times table of a nother number
Number bonds - 2 numbers that add together to make a given total, e.g. 8 and 2 bond to 10 or 73 and 27 bond to 100
Number line - a line either with numbers or without (a blank numberline). The number line emphasises the continuous nature of numbers and the existence of 'in-between' numbers that are not whole. It is based around the gaps between numbers. Children use this tool to help them count on or count back for addition of subtraction. As they get older, children will count in 'jumps' on a number line e.g. to add 142 to a number they may 'jump' 100 and then 40 and then 2. The number line is sometimes used in multiplication and division but can be time consuming.
Number track - a sequence of numbers, each inside its own square. It is a simplified version of the number line that emphasises the whole numbers.

Numicon - practical maths equipment that teacheschildren the names and values of numbers 1-10 initia lly but them helps them with early a ddition, subtraction, multiplic ation and division. Numic on is useful for showing the real value of a number practically.
One-Step Calculation - a calculation involving only one operation e.g. addition. Usually the child must decide what that operation is.
Partition - split up a larger number into parts, such as the hundreds, tens and unitse.g. 342 can be partitioned into 300 and 40 and 2
Place Value - the value of a digit created by its position in a numbere.g. 3 represents thirty in 234 but three thousand in 3567
Recombine - for addition, once you have partitioned numbers into hundreds, tens and units then you have to add then hundredstogether, then add the tens to that total, then add the units to that total
Remainder-a whole number left over after a division calculation
Repeated addition - repeatedly adding groups of the same size for multiplic ation
Scaling - an approach to multiplication whereby the number is 'scaled up' by a factor of the multipliere.g. $4 \times 3$ means 4 scaled up by a factor of 3.
Sharing - an approach to division whereby the dividend is shared out into a given number of groups (like dealing cards)
Short Division - traditional method for division with a single digit divisor (this is a compact version of long division, sometimes called 'bus stop')
Significant digit- the digit in a number with the largest value e.g. in 34 the most signific ant digit is the 3 , as it has a value of ' 30 ' and the ' 4 ' only has a value of ' 4 '
Single digit - a number with only one digit. These are always less than 10.
Sum - the total of two or more numbers (it implies addition). Sum should not be used as a synonym forcalculation.
Two-step calculation - a calculation where two different operations must be applied e.g. to find change in a shop you will usually have to add the individual prices and then subtract from the total amount. Usually the child has to decide what these two operations are and the order in which they should be applied.

